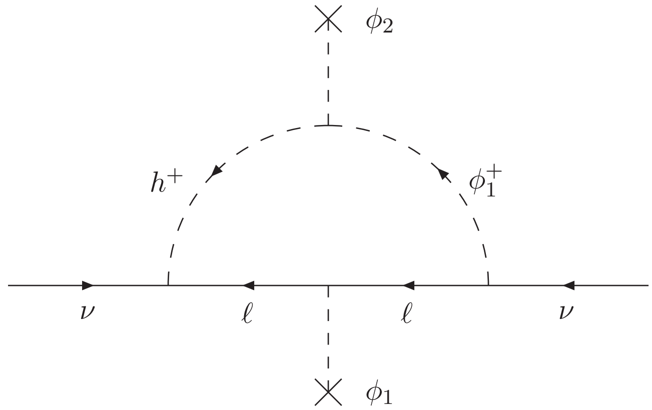


Fig.(1): One loop contribution to neutrino mass in $\{h\phi_1\phi_2\}$ model.

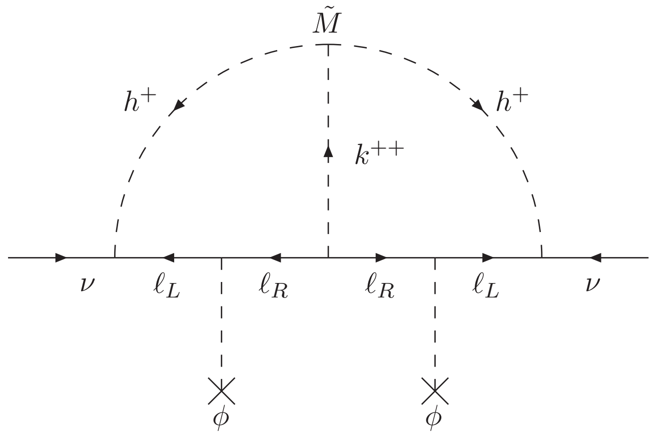


Fig.(2): Two loop contribution to neutrino mass in $\{h\phi k\}$ model.

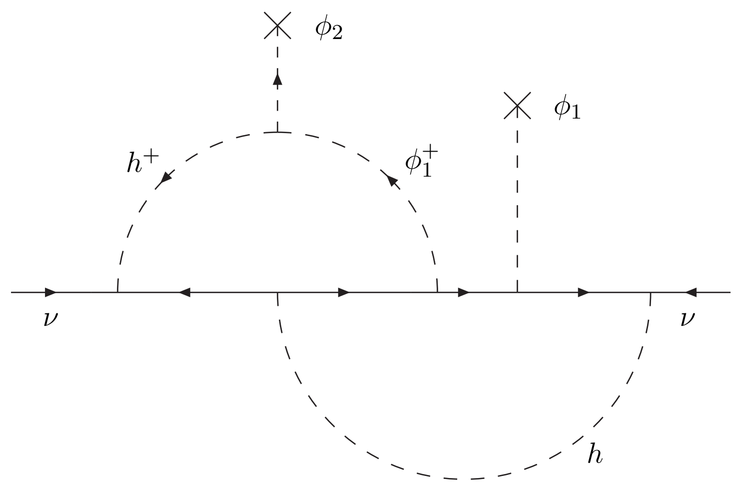


Fig.(3a): Two loop contribution to neutrino mass in $\{h\phi_1\phi_2\}$ model.

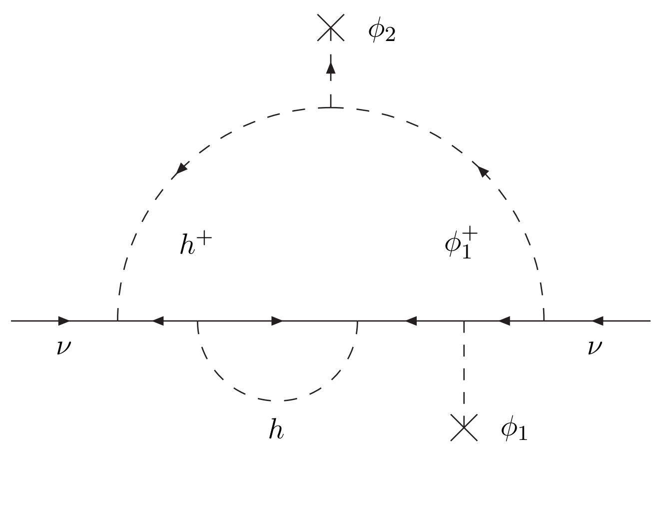

Fig.(3b): Two loop contribution to neutrino mass in $\{h\phi_1\phi_2\}$ model.

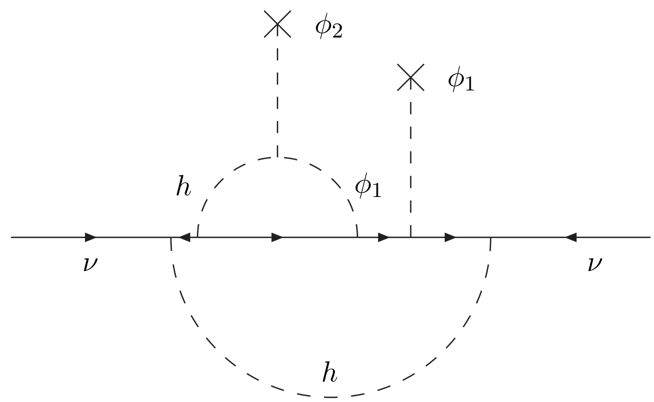


Fig.(3c): Two loop contribution to neutrino mass in $\{h\phi_1\phi_2\}$ model.

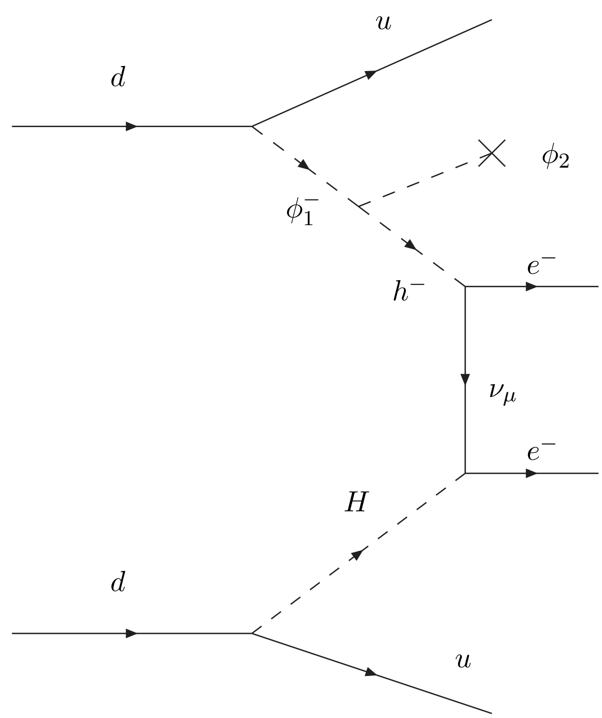


Fig.(4a): Tree level contribution to neutrinoless double beta decay in $\{h\phi_1\phi_2\}$ model. $H$ is the physical charged Higgs from the doublets.

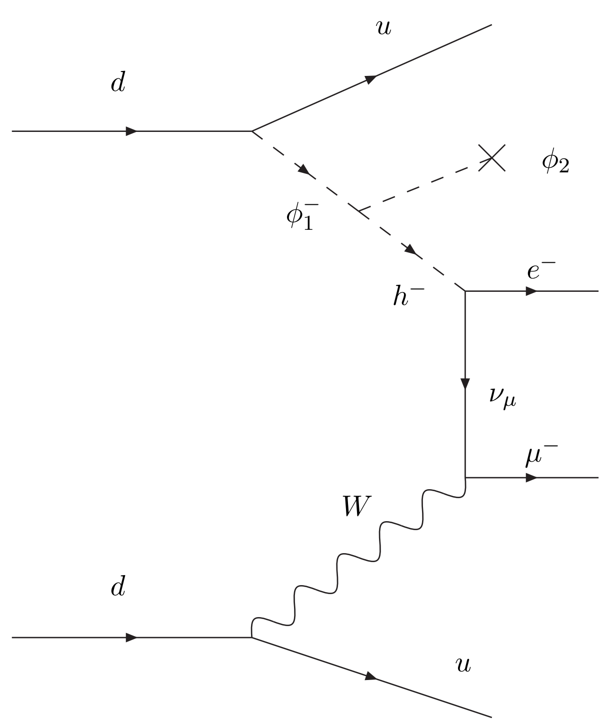


Fig.(4b): Tree level contribution to $\mu^-$–$e^+$ conversion in nuclei in $\{h\phi_1\phi_2\}$ model.

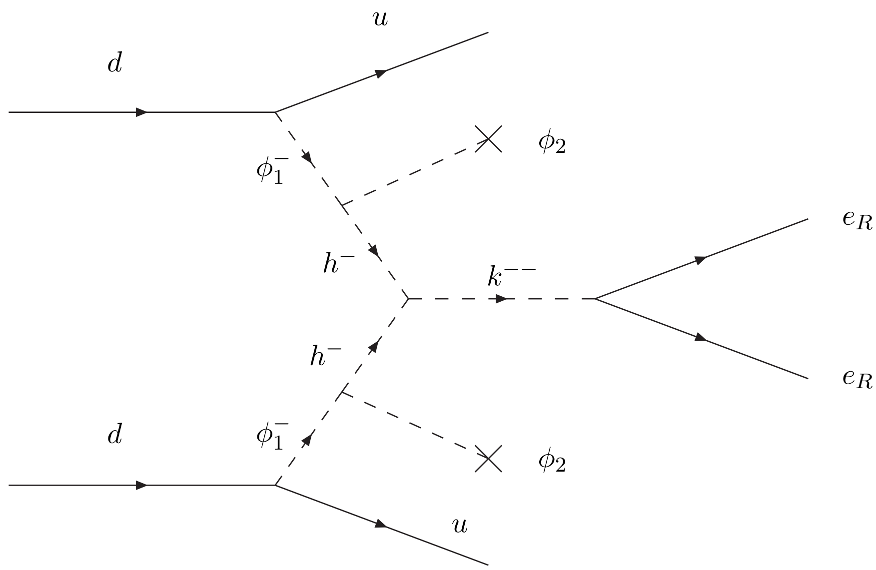


Fig.(5): Potential tree level contribution to neutrinoless double beta decay in $\{h\phi_1\phi_2 k\}$ model.

# Radiatively Induced Neutrino Majorana Masses and Oscillation

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## Abstract

We review and remark on models of radiatively induced neutrino Majorana masses and oscillations. It is pointed out that while the models are capable of accounting for the observed solar and atmospheric neutrino oscillation, some of them can also induce neutrinoless double beta decay and $\mu^- - e^+$ conversion in nuclei large enough to be potentially observable in the near future.


With the recent results of the SuperKamiokande experiments and other experiments [1], the field of neutrino oscillation and masses has moved from idle speculation to hard science. Let us ask what is the simplest modification one can make to the standard model in order to have neutrino oscillation and masses. Almost twenty years ago, one of us asked precisely this question and arrived at a model of lepton number violation and neutrino Majorana masses [2]. Some variations on this model [3,4] were considered a few years later. The various versions of this model has been studied over the years, [5,7,6,8] and recently has attracted a great deal of attention. [10–14] The purpose of this paper is to make some further remarks on this class of models. We start with a review of the model and its variations, so that this paper is largely self contained.

In the standard model, the neutrinos are of course massless. We will follow the old fashioned practice of economy, and try to add as little theoretical structure as possible. We can introduce either a Dirac mass or a Majorana mass for the neutrino. In the first alternative, one needs to introduce right handed neutrino fields and the question immediately arises on why the neutrino Dirac masses are so small compared to the charged lepton masses in the theory. This question was answered elegantly by the see-saw mechanism, in which the right handed neutrino fields are given large Majorana masses. But if we are willing to introduce Majorana masses to the right handed neutrino fields, perhaps we should consider dispensing with right handed neutrino fields altogether and simply try to generate Majorana masses for the existing left handed neutrino fields. We shall try to generate this Majorana

masses through quantum mechanical effect. This has the added advantage of having naturally small neutrino mass instead of relying on a new heavy scale for this explanation in the seesaw scheme.

Since in the standard model, the left handed neutrino fields belong to doublets $\psi_{aL} = \begin{pmatrix} \nu_a \\ l_a \end{pmatrix}_L$ (with $a$ a family index) we cannot simply put in Majorana mass terms. The general philosophy followed in Ref. [2–4] is that we should feel freer to alter the scalar field sector than other sectors since the scalar field sector is the least established one in the standard model. Out of the doublets we can form the Lorentz scalar $(\psi^i_{aL} C \psi^j_{bL})$ (where $i, j$ denote electroweak $SU(2)$ indices and $C$ the charge conjugation matrix): this can be either a triplet or a singlet under $SU(2)$. If we couple a triplet field to this lepton bilinear, then when the neutral component of the triplet field acquires a vacuum expectation value, the neutrinos immediately acquire Majorana masses. We considered this model unattractive: not only does it lack predictive power, but the rather accurately studied ratio of $W$ and $Z$ boson masses puts a stringent bound on any triplet Higgs. In addition, there is no natural way to explain the smallness required of this vacuum expectation value. We thus chose the alternative of coupling to an $SU(2)$ singlet (charged) field $h^+$ via the term $f^{ab}(\psi^i_{aL} C \psi^j_{bL})\varepsilon_{ij} h^+$.

An interesting point is that due to Fermi statistics the coupling $f^{ab}$ must be antisymmetric in $a$ and $b$. We are forced to couple leptons in one family to leptons in another one. Thus, the term above contains $f^{e\mu}(\nu_e C \mu^- - e^- C \nu_\mu)h^+$, for instance. As we will see, this leads to an interesting texture in the resulting neutrino mass matrix.

The term $f^{ab}(\psi^i_{aL} C \psi^j_{bL})\varepsilon_{ij} h^+$ in itself does not violate lepton number $L$ since we can always assign $L = -2$ to $h^+$. But we note that we can also couple $h^+$ to the Higgs doublets via $M_{\alpha\beta}\phi_\alpha\phi_\beta h^+$ if there are more than one Higgs doublet. By Bose statistics, the coupling matrix $M_{\alpha\beta}$ is antisymmetric. If the doublets are required to have zero lepton number by their respective Yukawa couplings, lepton number is now violated by two units, just right for generating neutrino Majorana masses.

We do not regard the necessity of more than one Higgs doublet as unattractive. Indeed,

theorists have always been motivated by one reason or another to introduce additional Higgs doublets. For example, some of the more attractive theories of CP violation [15] requires more than one Higgs doublets.

We refer to this class of models as $\{h\phi_1\phi_2\}$. From general principles we know that neutrino Majorana masses must be generated and that they must come out as finite, that is calculable in terms of the parameters of the theory. Indeed, we see that calculable neutrino Majorana masses are generated by the one loop diagram in Fig.(1).

To see finiteness, one way is to simply count the number of propagators. Another way, as is well-known, is to note that in this model, $h^-$ and the negatively charged components of $\phi_1$ and $\phi_2$ mix. After one component is eaten by the $W^-$ the contribution of the other two components to the one loop diagram in Fig.(1) cancel in a scalar version of the Glashow-Iliopoulos-Maiani mechanism.

Whenever more than one Higgs doublets are present, we have to worry about flavor changing effects from Higgs exchange. In the literature, there is of course no lack of mechanism to suppress this flavor changing effects. In the present context, a particularly clean proposal is that of Wolfenstein [5], who imposed a discrete symmetry so that one of the two Higgs doublets, say $\phi_2$, does not couple to leptons. By assigning $L = 2$ to $\phi_2$ we conserve lepton number. At this point, there are two possibilities. We can break lepton number softly by a term $m_{12}^2\phi_1^\dagger\phi_2 + H.c.$ or we can break lepton number spontaneously by having $\phi_2$ acquire a vacuum expectation value and create a Majoron. Since the doublet Majoron model is already ruled out by LEP data, we will assume the symmetry is softly broken.

In any case, we will suppose that some unspecified mechanism suppresses flavor changing effects from Higgs exchange, so that the effective Higgs couplings in Fig.(1) do not change flavor. Then we see that this one-loop diagram gives a neutrino mass matrix with the texture

$$(m_\nu)_{ab} = cf_{ab}(m_a^2 - m_b^2) = c([f, m^2])_{ab}, \tag{1}$$

where $c$ is $a_1(M_{12}/M_h^2)\ln(m_\phi^2/M_h^2)$ and $a_1$ is of order of the one-loop factor $1/(16\pi^2)$. In particular, the diagonal elements vanish. Various authors [11,12] have used this texture to

fit the atmospheric and solar neutrino data (but not the LSND data). They found that the data can be accommodated if $f_{\mu\tau} << f_{e\tau} << f_{e\mu}$, and $f_{e\tau}m_\tau^2 \sim f_{e\mu}m_\mu^2$. In that case, the solar neutrino oscillation would be due to either large angle MSW or vacuum oscillation. The neutrino mixing matrix has the form

$$U^\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

and $cf_{e\mu}m_\mu^2 = cf_{e\tau}m_\tau^2 = \sqrt{\Delta M_{atm}^2/2} = (1.6\text{–}5.5) \times 10^{-2} eV$. The coupling $f_{\mu\tau}$ can be related to $f_{e\tau}$ through $f_{\mu\tau} = f_{e\tau}(\Delta M_{solar}^2/\Delta M_{atm}^2)$. According to the recent data [1] $\Delta M_{solar}^2 = 1.8 \times 10^{-5} eV^2$ with $\sin^2\theta = 0.76$ if one take the large angle MSW solution while $\Delta M_{solar}^2 = 6.5 \times 10^{-11} eV^2$ with $\sin^2\theta = 0.75$ for vacuum oscillation solution. The same experiment also found $\Delta M_{atm}^2 = (0.5\text{–}6) \times 10^{-3} eV^2$ with $\sin^2\theta > .82$) for atmospheric neutrino oscillation.

The observed hierarchy of couplings ( $f_{\mu\tau} << f_{e\tau} << f_{e\mu}$) may indicate the approximate conservation of the additive quantum number $L_\mu + L_\tau - L_e$, as we will discuss below.

It is worth emphasizing that the texture in (1) is the result of the model plus our assumption that flavor-changing Higgs couplings are suppressed. If we relax the latter assumption, then $(m_\nu)_{ab}$ would be a general 3 by 3 symmetric matrix.

The phenomenological consequences of introducing the field $h$ have been well studied [2,3,9] and so we will not go into it here. We will merely note one particularly attractive feature [3]. Probably one of the best measured processes in leptonic weak interatcion is muon decay $\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$ to which $h$ exchange contributes at tree level. Fortunately, $h$ couples to left-handed fields and so upon Fierz rearrangement we see that only the overall magnitude, but not the angular distribution, of muon decay is affected. This sets a relatively weak bound on $f^2/m_h^2$.

Some years after this model was proposed, one of us [4] noted that in line with the same philosophy we can also introduce a doubly charged field $k^{++}$ coupling to the right-handed lepton singlets, thus $\tilde{f}_{ab} l_{aR} C l_{bR} k^{++}$. Fermi statistics now requires that $\tilde{f}$ be symmetric.

Lepton number is broken by the trilinear coupling $\tilde{M}k^{++}h^-h^-$. One therefore does not need to introduce additional doublet. The two-loop diagram in Fig.(2) generates a two-loop contribution to the neutrino Majorana matrix of the form

$$(m_\nu^{(2)})_{ab} = c_2 \sum_{cd} f_{ac}\tilde{f}_{cd}f_{db}\tilde{M}(m_c m_d/M_k^2) \tag{2}$$

where $c_2$ is a two-loop factor $c_2 = [ln(M_k^2/M_h^2 + 1)]^2/(32\pi^4)$ [7]. We refer to this as the $\{h\phi k\}$ model. One of course can also have additional doublets and the associated one loop contribution to neutrino mass. We refer to this as the $\{h\phi_1\phi_2 k\}$ model. In this case the one-loop and the two-loop contributions are to be added.

Later, Babu [7] studied these two classes of models. In particular, in the $\{h\phi k\}$ model we have $Det[m^{(2)}] = 0$ since $Det[f] = 0$ due to the fact that $f$ is a $3 \times 3$ antisymmetric matrix.

At this point we depart from our general treatment and focus on specific possibilities as suggested by experiments. The various possible textures of the Majorana neutrino mass matrix, in particular those dictated by the conservation of additive combination of electron, muon, and tau numbers, such as $L_\mu + L_\tau - L_e$, were studied in [4]. Typically, it is awkward in many models of neutrino masses to impose conservation of these additive combinations, however, as was pointed out in [4], it can be quite naturally implemented in this class of models by simply setting various couplings to zero. For example, suppose we set $f_{\mu\tau}$ to zero. Then the $f_{e\mu}$ term demands that the $h$ field to carry $L_e = -1, L_\mu = -1$, and $L_\tau = 0$, while the $f_{e\tau}$ term demands that the $h$ field to carry $L_e = -1, L_\mu = 0$, and $L_\tau = -1$. The clash between these two terms implies that $L_\mu$ and $L_\tau$ are violated, but that $L_\mu + L_\tau - L_e$ and $L_e$ are conserved.

One purpose of this paper is to study the two-loop contribution to $m_\nu$ in the $\{h\phi_1\phi_2\}$ model. The relevant diagrams are shown in Fig.(3). For instance, the diagram in Fig.(3a) contributres to $(m_\nu)_{ab}$ a term

$$(m_\nu^{(2)})_{ab} = \gamma \sum_{c,d} f_{ac}f^*_{cd}f_{db}(m_c^2 - m_d^2) = \gamma(f[m^2, f^*]f)_{ab} \tag{3}$$

where $\gamma = a_2(16\pi^2)^{-2}(M_{12}/M_h^2)$ with $a_2$ of order one. We are interested in the diagonal entries in $(m_\nu^{(2)})_{ab}$ since the off-diagonal entries just give a small perturbation to the one-loop contribution in (1). We see from the antisymmetry of $f$ that the diagonal elements necessarily involve the product of all three of the non-zero $f_{ab}$, $(a \neq b)$. Thus, for instance, $(m_\nu^{(2)})_{ee} = \gamma f_{e\tau} f^*_{\tau\mu} f_{\mu e}(m_\tau^2 - m_\mu^2) \sim \gamma f_{e\tau} f^*_{\tau\mu} f_{\mu e} m_\tau^2$. Note that similarly $(m_\nu^{(2)})_{\mu\mu} \sim \gamma f^*_{e\tau} f_{\tau\mu} f_{\mu e} m_\tau^2$ which is equal to $(m_\nu^{(2)})_{ee}$. An interesting texture emerges upon noting that $(m_\nu^{(2)})_{\tau\tau}$ is smaller by a factor $m_\mu^2/m_\tau^2$.

Thus, for phenomenological analysis we have a neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} r & a & b \\ a & s & c \\ b & c & t \end{pmatrix}$$

with the texture $a \sim b >> c > r \sim s >> t$. We expect that the terms $r \sim s$ would provide small corrections to the phenomenological analysis of Jarlskog et al [11].

It is probably premature to consider the effects of CP violation; the enormous difficulty of measuring CP violation in neutrino oscillations has been discussed recently [17]. But it may still be interesting to investigate the potential source of CP violation in various models. For the Yukawa coupling of $h^+$, assuming that the charged lepton mass matrix has been diagonalized, it is possible to redefine the phases of $\psi_a$ such that $f_{ab}$ is a real matrix for three generation case. Therefore, in the $\{h\phi_1\phi_2\}$ model, additional CP violation has to come from the Yukawa coupling of $\phi_2$. If $\phi_2$ does not couple to fermions, due to, say, softly broken lepton number symmetry, there will be no new explicit CP violating phase in the theory. Even the phase in the soft breaking term $m_{12}^2\phi_1^\dagger\phi_2 + H.c$ can be absorbed. The story is of course changed if both $\phi_1, \phi_2$ are allowed to to have Yukawa couplings. On the other hand, in the $\{h\phi k\}$ model, the phases in the Yukawa coupling $\tilde{f}$ can not longer be absorbed. Therefore, there will be CP violation in the two loop mass matrix in Eq.(2).

While our discussion is not supersymmetric, however as a mechanism it can be easily imbedded into a supersymmetric theory [16]. In particular the $h^+$ has exactly the gauge

quantum number of a right-handed electron. Therefore if we allow lepton number violating R-parity breaking terms, the role of the term $f^{ab}(\psi^i_{aL}C\psi^j_{bL})\varepsilon_{ij}h^+$ in the $\{h\phi_1\phi_2\}$ model can be played exactly by part of such R-parity breaking terms in which $h^+$ is the right-handed slepton and $\phi_i$ are linear combinations of the Higgs doublet and slepton doublets. A large portion of our discussions obviously applies to this class of models.

It was emphasized [8] some time ago that in some models even if the Majorana neutrino masses are naturally small due to their quantum origin, the associated neutrinoless double beta decay [18] may be a tree level effect and thus much larger than one would have naively guessed based on the small masses. In fact some of the models discussed here naturally exhibit this possibility. In the $\{h\phi_1\phi_2\}$ model, contrary to the claim in Ref [11], there is a potential tree level contribution to the neutrinoless double beta decay as shown in Fig.(4a) if both $\phi_i$ are allowed to couple to fermions. The estimate of the diagram is very similar to the ones in Ref [19]. The scalar $h$ induces a four fermion interaction $\epsilon_1^{\mu e}(4G_F/\sqrt{2})\bar{d}_R u_L \nu_\mu^T C^{-1} e^L$, using the notation in Ref. [19], with $\epsilon_1^{\mu e}$ estimated to be $(4G_F/\sqrt{2})^{-1} f_{e\mu} h_d v_2 M_{12}/(M_h^2 M_\phi^2)$ where $h_d$ is the Yukawa coupling of the down quark to $\phi_1$. One also needs the exchange of the physical charged Higgs boson $H^+$ in order to change $\nu_\mu$ back into electron in Fig.(4a). We parameterize this $H^+$ exchange amplitude by taking its ratio with the $W$ exchange amplitude as in $r_{e\mu} = (\tilde{h}_{du}\tilde{h}_{e\mu}/M_H^2)(4G_F/\sqrt{2})^{-1}$ where $\tilde{h}_{ij}$ are the coupling constants of $H^+$. The current experimental constraint is $r_{e\mu}\epsilon_1^{\mu e} < 1 \times 10^{-8}$. Taking $cf_{e\mu}m_\mu^2 = 4 \times 10^{-2} eV$ from the neutrino data and $h_d v_2 \sim m_d \sim 6MeV$ and $M_\phi = 100GeV$, one obtains the limit $r_{e\mu}(1.6 \times 10^{-4}) < 1 \times 10^{-8}$. This gives rise to stringent constraint on the angle $r_{e\mu}$. Similar charged Higgs exchange can also give rise to $e-\tau$ flavor mixing. The corresponding parameter, $r_{e\tau}$, is constrained to be $r_{e\tau}(1.6 \times 10^{-4})(m_\mu/m_\tau)^2 < 1 \times 10^{-8}$. Note that if one uses $W$ exchange instead of $H^+$ exchange, the tree level diagram cannot mediate neutrinoless double beta decay. However, the diagram in Fig.(4b) can give rise to observable $\mu^-$–$e^+$ conversion [20] in nuclei as pointed out in Ref. [19]. In the $\{h\phi k\}$ model, there is no tree level contribution to neutrinoless double beta decay. In the $\{h\phi_1\phi_2 k\}$ model, in addition to the contribution in Fig.(4a), there are additional tree level contributions

to neutrinoless double beta decay as shown in Fig.(5). The estimate of this diagram is similar to the one in Ref. [8]. The diagram gives rise to the operator $G_2(\bar{d}_R u_L)^2 e_R^T C^{-1} e_R$ with $G_2 \sim (h_d v_2 M_{12})^2 \tilde{f}_{ee} \tilde{M}/(M_h^4 M_\phi^4 M_k^2)$. Assuming that various contributions do not cancel each other, we find that the current experimental limit gives rise to the constraint $|(G_2 m_p/G_F^2) 4\pi \langle f|\Omega_\Delta|i\rangle\,| < 0.8 \times 10^{-4}$ where $\langle f|\Omega_\Delta|i\rangle$ is a dimensionless nucleus matrix element as defined in Ref. [8]. A rough estimate [8] of $4\pi \langle f|\Omega_\Delta|i\rangle$ is 200. For example, taking $M_h \sim M_\phi \sim M_k \sim M_{12} \sim \tilde{M} \sim 100 GeV$, this constrains $\tilde{f}_{ee} < 0.16$.

In conclusion, we have summarized some interesting models of radiative neutrino masses without any additional fermions added to the standard model. We emphasized that, in the simplest $\{h\phi_1\phi_2\}$ model, in addition to the one loop diagram, there are two loop diagrams that may give rise to interesting texture of neutrino mass for potential future experimental test. We point out that in some of these models, the neutrinoless double beta decay may be a tree level effect and, as a result, may give rise to serious contraint on the theory when combined with the neutrino oscillation data. In addition, the model may give rise to observable $\mu^-$–$e^+$ conversion in nucleus.

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